K-CET EXAMINATION – 2024

MATHEMATICS – A-3 with Key & Solutions

1. If $2\sin^{-1}x - 3\cos^{-1}x = 4$, $x \in [-1,1]$ then $2\sin^{-1}x + 3\cos^{-1}x$ is equal to

(A) $\frac{4-6\pi}{5}$ (B) $\frac{6\pi-4}{5}$

(C) $\frac{3\pi}{2}$ (D) 0

Ans. (B)

Sol. $2\sin^{-1}x - 3\left(\frac{\pi}{2} - \sin^{-1}x\right) = 4$

$2\sin^{-1}x - \frac{3\pi}{2} + 3\sin^{-1}x = 4$

$5\sin^{-1}x = 4 + \frac{3\pi}{2}$

$\sin^{-1}x = \frac{4}{5} + \frac{3\pi}{10}$

$2\sin^{-1}x + 3\cos^{-1}x$

$= 2\sin^{-1}x + 3\left(\frac{\pi}{2} - \sin^{-1}x\right)$

$= \frac{3\pi}{2} - \sin^{-1}x$

$= \frac{3\pi}{2} - \left(\frac{4}{5} + \frac{3\pi}{10}\right)$

$= \frac{3\pi}{2} - \frac{3\pi}{10} - \frac{4}{5} = \frac{12\pi}{10} - \frac{4}{5}$

$= \frac{6\pi - 4}{5}$

2. If A is a square matrix such that $A^2 = A$, then $(I + A)^3$ is equal to

(A) 7A – I

(B) 7A

(C) 7A + I

(D) I – 7A

Ans. (C)

Sol. $(I + A)^3 = (I + A)^2 (I + A)$

$(I^2 + A^2 + 2IA)(I + A)$

$= I^3 + I^2A + A^2I + A^3 + 2I^2A + 2IA^2$

$= I + A + A + A^2 . A + 2A + 2A^2$

$= I + A + A + A. A + 2A + 2A$

$= I + A + A + A^2 + 4A$

$= I + 2A + A + 4A$

$= 7A + I$

3. If $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, then A^{10} is equal to

(A) $2^8 A$ (B) $2^9 A$

(C) $2^{10} A$ (D) $2^{11} A$

Ans. (B)

Sol. $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = A^2 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} = 2^1 A$

$A^3 = A^2. A = (2A) A = 2A^2 = 2(2A) = 4A = 2^2 A$

$\therefore A^{10} = 2^9 A$

4. If $f(x) = \begin{vmatrix} x-3 & 2x^2-18 & 2x^3-81 \\ x-5 & 2x^2-50 & 4x^3-500 \\ 1 & 2 & 3 \end{vmatrix}$, then f(1). f(3) + f(3). f(5) + f(5). f(1) is

(A) –1 (B) 0

(C) 1 (D) 2

Ans. None of the option is correct

Sol. Question is wrong in the place of $2x^3 - 81$ it should be $3x^3 - 81$ then option will be (B)

5. Let (gof) (x) = sin x and (fog) (x) $= (\sin\sqrt{x})^2$. then

(A) $f(x) = \sin^2 x$, $g(x) = x$

(B) $f(x) = \sin\sqrt{x}$, $g(x) = \sqrt{x}$

(C) $f(x) = \sin^2 x, g(x) = \sqrt{x}$

(D) $f(x) = \sin\sqrt{x}$, $g(x) = x^2$

Ans. (C)

Sol. $f(x) = \sin^2 x, g(x) = \sqrt{x}$

6. Let A = {2, 3, 4, 5,……16, 17, 18}. Let R be the relation on the set A of ordered pairs of positive integers defined by (a, b) $\mathbb{R}$ (c, d) if and only if ad = bc for all (a, b), (c, d) in A × A. Then the number of ordered pairs of the equivalence class of (3, 2) is

(A) 4 (B) 5

(C) 6 (D) 7

Ans. (C)

Sol. (a, b) R (c, d) ⇒ ad = bc

$\Rightarrow \frac{a}{b} = \frac{c}{d}$

$\frac{3}{2} = \frac{6}{4} = \frac{9}{6} = \frac{12}{8} = \frac{15}{10} = \frac{18}{12}$

7. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then x (y + z) + y (z + x) + z (x + y) equals to

(A) 0 (B) 1
(C) 6 (D) 12

Ans. (C)

Sol. Let x = –1, y = –1, z = –1

x (y + z) + y (z + x) + z (x + y) = 2 + 2 + 2 = 6

8. The function f(x) = |cos x| is

(A) Everywhere continuous and differentiable
(B) Everywhere continuous but not differentiable at odd multiples of $\frac{\pi}{2}$
(C) Neither continuous nor differentiable at $(2n+1)\frac{\pi}{2}, n \in Z$
(D) Not differentiable everywhere

Ans. (B)

Sol. f(x) = |cos x|

Not differentiable at cos x = 0 ⇒ x = (2x +1) $\frac{\pi}{2}, n \in z$

f(x) is not differentiable at odd multiple of $\frac{\pi}{2}$ ∴ (B) is correct

9. If $y = 2x^{3x}$, then $\frac{dy}{dx}$ at x = 1 is

(A) 2 (B) 6
(C) 3 (D) 1

Ans. (B)

Sol. $y = 2x^{3x}$

Logy = log2 + 3x logx

$\frac{1}{y}\frac{dy}{dx} = 0 + \frac{3x}{x} + 3\log x$

$\frac{dy}{dx} = y\,(3+3\log x)$

$= 2x^{3x}\,(3+3\log x)$

$\left(\frac{dy}{dx}\right)_{x=1} = 2(1)\,(3+3\log 1)$

= 2 (3+0)

= 6

10. Let the function satisfy the equation f(x + y) = f(x) f(y) for all $x, y \in \mathbb{R}$, where f(0) ≠ 0. If f(5) = 3 and $f'(0) = 2$ then $f'(5)$ is

(A) 6 (B) 0
(C) 5 (D) –6

Ans. (A)

Sol. f (x + y) = f(x) f(y)

Let $f(x) = a^x$

$f(5) = 3 \Rightarrow a^5 = 3 \rightarrow (1)$

$f^1(x) = a^x \log a$

$\therefore\ 2 = f^1(0) = a^0 \log a = \log a$

$\Rightarrow\ 2 = \log a$

$f^1(5) = a^5 \log a = (3)(2) = 6$

11. The value of C in (0, 2) satisfying the mean value theorem for the function $f(x) = x(x-1)^2,\ x \in [0,2]$ is equal to

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$
(C) $\frac{1}{3}$ (D) $\frac{2}{3}$

Ans. (C)

Sol. $f^1(x) = x\,(2(x-1)) + (x-1)^2\,(1)$

$= 2x\,(x-1) + (x-1)^2$

$f^1(c) = 0 \Rightarrow 2c\,(c-1) + (c-1)^2 = 0$

$\Rightarrow (c-1)\,[2c+c-1] = 0$

$\Rightarrow C=1,\ c = \frac{1}{3}$

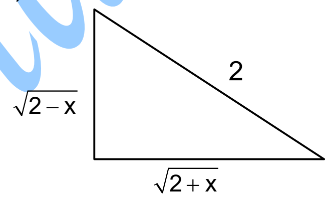
$\therefore\ c = \frac{1}{3} \in (0, 2)$

12. $\frac{d}{dx}\left[\cos^2\left(\cot^{-1}\sqrt{\frac{2+x}{2-x}}\right)\right]$ is

(A) $-\frac{3}{4}$ (B) $-\frac{1}{2}$
(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Ans. (D)

Sol.



$\frac{d}{dx}\left(\cos^2 \cos^{-1}\left(\frac{\sqrt{2+x}}{2}\right)\right)$

$= \frac{d}{dx}\left(\frac{\sqrt{2+x}}{2}\right)^2$

$= \frac{d}{dx}\left(\frac{2+x}{4}\right) = \frac{d}{dx}\left(\frac{x}{4}+\frac{1}{2}\right) = \frac{1}{4}$

13. For the function $f(x) = x^3 - 6x^2 + 12x - 3$; x = 2 is

(A) a point of minimum
(B) a point of inflexion
(C) not a critical point
(D) a point of maximum

Ans. (B)

Sol. $f^1(x) = 3x^2 - 12x + 12$

$(f^1(x))_{x=2} = 12-24+12=0$

$(f^{11}(x))_{x=2} = 6x - 12 = 6(2)-12 = 0$

∴ a point of inflexion

14. The function X^x ; X > 0 is strictly increasing at

(A) $\forall x \in \mathbb{R}$ (B) $x < \frac{1}{e}$
(C) $x > \frac{1}{e}$ (D) $x < 0$

Ans. (C)

Sol. $f(x)=x^x$

$f'(x)=x^x(1+\log x)$

$f'(x)>0$

$x^x(x+\log x)>0$

$1+\log x>0$

$\log x>-1$

$x>e^{-1}$

$x>\frac{1}{e}$

15. The maximum volume of the right circular cone with slant height 6 units is

(A) $4\sqrt{3}\pi$ cubic units

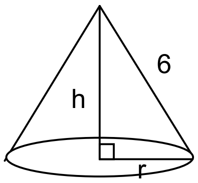
(B) $16\sqrt{3}\pi$ cubic units

(C) $3\sqrt{3}\pi$ cubic units

(D) $6\sqrt{3}\pi$ cubic units

Ans. (B)

Sol. $h^2+r^2=36$



$r^2=36-h^2$

$V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi(36-h^2)h$

$V=\frac{1}{3}\pi(36h-h^3)$

$f(h)=\frac{1}{3}\pi(36h-h^3)$

$f'(h)=\frac{\pi}{3}(36-3h^2)=\pi(12-h^2)$

$f''(h)=-2\pi h$

$f'(h)=0\Rightarrow 12-h^2=0$

$h^2=12$

$h=2\sqrt{3}$

Max Value = $f(2\sqrt{3})=\frac{\pi}{3}\left(36\ 2\sqrt{3}-(2\sqrt{3})^3\right)$

$=\frac{\pi}{3}(72\sqrt{3}-24\sqrt{3})=\frac{\pi}{3}(48\sqrt{3})=16\sqrt{3}\pi$

16. If $f(x) = X e^{x(1-x)}$ then f(x) is

(A) Increasing in $\mathbb{R}$

(B) Decreasing in $\mathbb{R}$

(C) Decreasing in $\left[-\frac{1}{2},1\right]$

(D) Increasing in $\left[-\frac{1}{2},1\right]$

Ans. (D)

Sol. $f(x)=xe^{x(1-x)}$

$f'(x)=xe^{x(1-x)}\left[x(-1)+1-x\right]+e^{x(1-x)}$

$f'(x)=xe^{x(1-x)}\left[-x+1-x\right]+e^{x(1-x)}$

$f'(x)=xe^{x(1-x)}\left[1-2x\right]+e^{x(1-x)}$

$f'(x)=xe^{x(1-x)}\left(x-2x^2+1\right)$

$f'(x)\geq 0$

$x-2x^2+1\geq 0$

$2x^2-x-1\leq 0$

$(x-1)(2x+1)\leq 0$

$(x-1)\left(x+\frac{1}{2}\right)\leq 0$

Increasing $x\in\left[-\frac{1}{2},1\right]$

17. $\int\frac{\sin x}{3+4\cos^2 x}dx=$

(A) $-\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)+C$

(B) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\cos x}{3}\right)+C$

(C) $\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{\cos x}{3}\right)+C$

(D) $-\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{3}\right)+C$

Ans. (A)

Sol. $\int\frac{\sin x}{3+4\cos^2 x}dx$

Put cosx = t,

–sinx dx = dt

sin dx = –dt

$\int\frac{-dt}{3+4t^2}$

$-\frac{1}{4}\int\frac{dt}{t^2+\frac{3}{4}}$

$-\frac{1}{4}\int\frac{dt}{t^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\frac{-1}{2\sqrt{3}}\tan^{-1}\left(\frac{2t}{\sqrt{3}}\right)+C$

$=\frac{-1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)+C$

18. $\int_{-\pi}^{\pi}(1-x^2)\sin x.\cos^2 x\,dx=$

(A) $\pi-\frac{\pi^2}{3}$ (B) $2\pi-\pi^3$

(C) $\pi-\frac{\pi^3}{3}$ (D) 0

Ans. (D)

Sol. $\int_{-\pi}^{\pi}(1-x^2)\sin x\cos^2 x\,dx$

$f(x)=(1-x^2)\sin x\cdot\cos^2 x$

$f(-x)=(1-x^2)(-\sin x)\cos^2 x$

$f(-x) = -(1-x^2)\sin x \cdot \cos^2 x$

$f(-x) = -f(x)$, f(x) is odd function

19. $\int \frac{1}{x\left[6(\log x)^2 + 7\log x + 2\right]}dx =$

(A) $\frac{1}{2}\log\left|\frac{2\log x+1}{3\log x+2}\right| + C$

(B) $\log\left|\frac{2\log x+1}{3\log x+2}\right| + C$

(C) $\log\left|\frac{3\log x+2}{2\log x+1}\right| + C$

(D) $\frac{1}{2}\log\left|\frac{3\log x+2}{2\log x+1}\right| + C$

Ans. (B)

Sol. $\int \frac{dx}{x\left(6(\log x)^2 + 7\log x + 2\right)}$

Put log x = t, $\frac{1}{x}dx = dt$

$\int \frac{dt}{6t^2+7t+2}$

$\int \frac{dt}{6t^2+4t+3t+2}$

$\int \frac{dt}{2t(3t+2)+1(3t+2)} = \int \frac{dt}{(3t+2)(2t+1)}$

$= \int\left(\frac{-3}{3t+2} + \frac{2}{2t+1}\right)dt$

$= \frac{-3}{3}\log|3t+2| + \frac{2}{2}\log 2t+1+c$

$= \log\left|\frac{2t+1}{3t+2}\right| + C = \log\left|\frac{2\log x+1}{2\log x+2}\right| + C$

20. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx =$

(A) $2x + \sin x + 2\sin 2x + C$

(B) $x + 2\sin x + 2\sin 2x + C$

(C) $x + 2\sin x + \sin 2x + C$

(D) $2x + \sin x + \sin 2x + C$

Ans. (C)

Sol. $\int \frac{2\sin\frac{5x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}dx$

$= \int \frac{\sin\left(\frac{5x+x}{2}\right) + \sin\left(\frac{5x-x}{2}\right)}{\sin x}dx$

$= \int \frac{\sin 3x + \sin 2x}{\sin x}dx$

$= \int \frac{3\sin x - 4\sin^3 x + 2\sin x\cos x}{\sin x}dx$

$= \int\left(3 - 4\sin^2 x + 2\cos x\right)dx$

$= \int\left(3 - 4\left(\frac{1-\cos 2x}{2}\right) + 2\cos x\right)dx$

$= \int\left(3 - 2 + 2\cos 2x + 2\cos x\right)dx$

$= \int\left(1 + 2\cos 2x + 2\cos x\right)dx$

$= x + 2\frac{\sin 2x}{2} + 2\sin x + c = x + \sin 2x + 2\sin x + c$

21. $\int_1^5 \left(|x-3| + |1-x|\right)dx =$

(A) 12 (B) $\frac{5}{6}$

(C) 21 (D) 10

Ans. (A)

Sol. $\int_1^3 (3-x)dx + \int_3^5 (x-3)dx + \int_1^5 (x-1)dx$

$\left[3x - \frac{x^2}{2}\right]_1^3 + \left[\frac{x^2}{2} - 3x\right]_3^5 + \left[\frac{x^2}{2} - x\right]_1^5$

$\left(9 - \frac{9}{2}\right) - \left(3 - \frac{1}{2}\right) + \left(\frac{25}{2} - 15\right) - \left(\frac{9}{2} - 9\right) + \left(\frac{25}{2} - 5\right) - \left(\frac{1}{2} - 1\right)$

= 25 – 9 – 4 = 25 – 23 = 12

22. $\lim_{n\to\infty}\left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + \ldots\ldots + \frac{1}{5n}\right) =$

(A) $\frac{\pi}{4}$ (B) $\tan^{-1}3$

(C) $\tan^{-1}2$ (D) $\frac{\pi}{2}$

Ans. (C)

Sol. $\lim_{n\to\infty}\sum_{i=1}^{2n}\frac{n}{n^2+i^2}$

$= \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{2n}\frac{1}{1+\left(\frac{i}{n}\right)^2}$

$= \int_0^2 \frac{dx}{1+x^2}$

$= \left[\tan^{-1}x\right]_0^2$

$= \tan^{-1}2$

23. The area of the region bounded by the line $y = 3x$ and the curve $y = x^2$ in sq. units is

(A) 10 (B) $\frac{9}{2}$

(C) 9 (D) 5

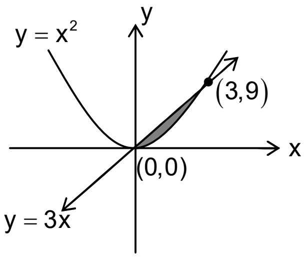
Ans. (B)

Sol. $3x = x^2$

$x^2 = 3x = 0$

$x(x-3) = 0$
$x = 0, 3$

Area = $\int_0^3 (3x - x^2)dx$



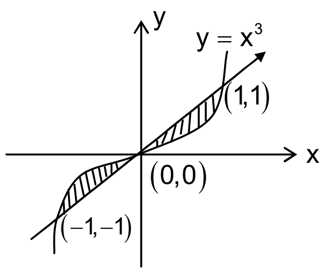
$= \left(\frac{3x^2}{2} - \frac{x^3}{3}\right)_0^3$

$= \frac{27}{2} - \frac{27}{3} \quad = 27\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{9}{2}$

24. The area of the region bounded by the line $y = x$ and the curve $y = x^3$ is
(A) 0.2 sq. units (B) 0.3 sq. units
(C) 0.4 sq. units (D) 0.5 sq. units

Ans. (D)

Sol. Area = $2\int_0^1 (x - x^3)dx$



$= 2\left(\frac{x^2}{2} - \frac{x^4}{4}\right)_0^1$

$= 2\left(\frac{1}{2} - \frac{1}{4}\right)$

$= 2 \times \frac{1}{4} = \frac{1}{2} = 0.5$

25. The solution of $e^{\frac{dy}{dx}} = x + 1, y(0) = 3$ is
(A) $y - 2 = x\log x = x$
(B) $y - x - 3 = x \log x$
(C) $y - x - 3 = (x + 1) \log (x + 1)$
(D) $y + x - 3 = (x + 1) \log (x + 1)$

Ans. (D)

Sol. $\frac{dy}{dx} = \log(x+1) \Rightarrow dy = \log(x+1)dx$

on integrating

$y = \int \log(1+x).1dx$

$y = \log(1+x).\ x - \int \frac{1}{1+x}.x.dx$

$y = x\log(1+x) - \int \frac{1+x-1}{1+x}dx$

$y = x\log(1+x) - \int 1dx + \log(1+x) + c$

$y = x\log(1+x) - x + \log(1+x) + c$

$\because y(0) = 3 \Rightarrow c = 3$

$\Rightarrow y + x - 3 = (x+1)\log(x+1)$

26. The family of curves whose x and y intercepts of a tangent at any point are respectively double the x and y coordinates of that point is
(A) $xy = C$
(B) $x^2 + y^2 = C$
(C) $x^2 - y^2 = C$
(D) $\frac{y}{x} = C$

Ans. (A)

Sol. $\frac{dy}{dx} = \frac{-2y}{2x}$

$\frac{dy}{dx} = \frac{-y}{x}$

$xdy + ydx = 0$

$d(xy) = 0$

$\Rightarrow xy = c$

27. The vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a ΔABC. The length of the median through A is
(A) $\sqrt{18}$ (B) $\sqrt{72}$
(C) $\sqrt{33}$ (D) $\sqrt{288}$

Ans. (C)

Ans. (C)

Sol. $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}, \quad \overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$

Length of the median $= \left|\frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}\right|$

$= \frac{1}{2}|8\hat{i} - 2\hat{j} + 8\hat{k}|$

$= \sqrt{33}$

28. The volume of the parallelopiped whose co–terminous edges are $\hat{j} + \hat{k},\ \hat{i} + \hat{k}$ and $\hat{i} + \hat{j}$
(A) 6 cu. units
(B) 2 cu. units
(C) 4 cu. units
(D) 3 cu. units

Ans. (B)

Sol. $\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 0 - 1(0-1) + 1(1-0)$

$= 0 + 1 + 1 = 2$

Option (B) is correct

29. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector if
(A) $\theta = \frac{\pi}{4}$
(B) $\theta = \frac{\pi}{3}$
(C) $\theta = \frac{2\pi}{3}$
(D) $\theta = \frac{\pi}{2}$

Ans. (C)

Sol. $|\vec{a}| = 1, |\vec{b}| = 1 \qquad (\vec{a}.\vec{b}) = \theta$

$|\vec{a} + \vec{b}| = 1$

$\Rightarrow 1 + 1 + 2(\vec{a} \cdot \vec{b}) = 1$

$\Rightarrow 2(\vec{a} \cdot \vec{b}) = -1$

$\Rightarrow \vec{a} \cdot \vec{b} = \frac{-1}{2} \Rightarrow \cos(\vec{a}, \vec{b}) = \frac{2\pi}{3}$

$\Rightarrow \theta = \frac{2\pi}{3}$

30. If $\vec{a}, \vec{b}, \vec{c}$ are three non – coplanar vectors and p,q,r are vectors defined by $\vec{p} = \frac{\vec{b}\times\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{q} = \frac{\vec{c}\times\vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}$

$\vec{r} = \frac{\vec{a}\times\vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$ then $(\vec{a}+\vec{b}).\vec{p} + (\vec{b}+\vec{c}).\vec{q} + (\vec{c}+\vec{a}).\vec{r}$ is

(A) 0 (B) 1
(C) 2 (D) 3

Ans. (D)

Sol. $(\vec{a}+\vec{b}).\vec{p} + (\vec{b}+\vec{c}).\vec{q} + (\vec{c}+\vec{a}).\vec{r}$

= 1 + 0 + 1 + 0 + 1 + 0 = 3

31. If lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are mutually perpendicular, then k is equal to

(A) $-\frac{10}{7}$ (B) $-\frac{7}{10}$
(C) –10 (D) –7

Ans. (A)

Sol. –3 (3K) + 2K (1) + 2(–5) = 0

– 9k + 2k = 10

–7k = 10

$\Rightarrow K = \frac{-10}{7}$

32. The distance between the two planes $2x+3y+4x=4$ and $4x+6y+8x=12$ is

(A) 2 units (B) 8 units
(C) $\frac{2}{\sqrt{29}}$ units (D) 4 units

Ans. (C)

Sol. 4x + 6y + 8z = 8

4x + 6y + 8z = 12

$\text{distance} = \frac{|12-8|}{\sqrt{4^2+6^2+8^2}}$

$= \frac{4}{\sqrt{16+36+64}} = \frac{4}{\sqrt{116}} = \frac{4}{\sqrt{4\times 29}} = \frac{2}{\sqrt{29}}$

33. The sine of the angle between the straight line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{4-z}{-5}$ and the plane $2x-2y+z=5$ is

(A) $\frac{1}{5\sqrt{2}}$ (B) $\frac{2}{5\sqrt{2}}$
(C) $\frac{3}{50}$ (D) $\frac{3}{\sqrt{50}}$

Ans. (A)

Sol. $\vec{r} = (2\hat{i}+3\hat{j}+4\hat{k}) + \lambda(3\hat{i}+4\hat{j}+5\hat{k})$

$\vec{r}.(2\hat{i}-2\hat{j}+\hat{k}) = 5$

$\sin\theta = \frac{6-8+5}{\sqrt{9+16+25}\sqrt{4+4+1}} = \frac{3}{\sqrt{50}\sqrt{9}} = \frac{1}{5\sqrt{2}}$

34. The equation xy = 0 in three–dimensional space represents

(A) A pair of straight lines
(B) A plane
(C) A pair of planes at right angles
(D) A pair of parallel planes

Ans. (C)

Sol. xy = 0

x = 0 or y = 0

yz-plane zx – plane

Pair of plane at right angle.

35. The plane containing the point (3, 2, 0) and the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$ is

(A) $x-y+z-=1$
(B) x + y + z = 5
(C) x + 2y – z = 1
(D) 2x – y + z = 5

Ans. (A)

Sol. By option verification, (3, 2, 0) and (3, b, 4) satisfies.

36. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let z = 4x + 6y be the objective function. The minimum value of z occurs at

(A) Only (0, 2)
(B) Only (3, 0)
(C) The mid – point of the line segment joining the points (0, 2) and (3, 0)
(D) Any point on the line segment joining the points (0, 2) and (3, 0)

Ans. (D)

Sol. Corner points

A= (0, 2)

B = (3, 0)

C = (6, 0)

D = (6, 8)

E = (0, 5)

Objective function Z = 4x + 6y

$Z_A = 4(0)+6(2) = 0+12 = 12$

$Z_B = 4(3)+6(0) = 12+0 = 12$

$Z_C = 4(16)+6(0) = 24+0 = 24$

$Z_D = 4(16)+6(8) = 24+48 = 72$

$Z_E = 4(0)+6(5) = 0+30 = 30$

Z has minimum at point A = (0, 2) and B = (3, 0)

37. A die is thrown 10 times. The probability that an odd number will come up at least once is

(A) $\frac{11}{1024}$ (B) $\frac{1013}{1024}$
(C) $\frac{1023}{1024}$ (D) $\frac{1}{1024}$

Ans. (C)

Sol. Random experiment F = Throwing a die 10 times

Event E: Getting odd number

$p(E)=\frac{3}{6}=\frac{1}{2}$

Probability of getting an odd number atleast one time = 1 – P(getting odd number zero times)

$=1-\left(\frac{1}{2}\right)^{10}=1-\frac{1}{2^{10}}=1-\frac{1}{1024}=\frac{1023}{1024}$

38. A random variable X has the following probability distribution:

X	0	1	2
P(X)	$\frac{25}{36}$	k	$\frac{1}{36}$

If the mean of the random variable X is $\frac{1}{3}$, then the variance is

(A) $\frac{1}{18}$ (B) $\frac{5}{18}$

(C) $\frac{7}{18}$ (D) $\frac{11}{18}$

Ans. (B)

Sol. Given:

X	0	1	2
P(X)	$\frac{25}{36}$	k	$\frac{1}{36}$

$[p(x)]=1$ and $E[x]=\frac{1}{3}$

$\frac{25}{36}+K+\frac{1}{36}=1$

$\frac{26}{36}+K=1$

$K=1-\frac{26}{36}=\frac{10}{36}$

$V[x]=E[x^2]-(E(x))^2$

$=\sum x_i^2 p(x_i)-\sum(x_i\,p(x_i))^2$

$=0^2.\frac{25}{36}+1^2.K+2^2.\frac{1}{36}-\left(\frac{1}{3}\right)^2$

$=K+4.\frac{1}{36}-\frac{1}{9}$

$=\frac{10}{36}+\frac{4}{36}-\frac{4}{36}=\frac{10+4-4}{36}=\frac{10}{36}=\frac{5}{18}$

39. If a random variable X follows the binomial distribution with parameters n = 5, p and P(X = 2) = 9P (X = 3), then p is equal to

(A) 10 (B) $\frac{1}{10}$

(C) 5 (D) $\frac{1}{5}$

Ans. (B)

Sol. Given: n = 5

p(x = 2) = 9. p(x = 3)

${}^5C_2.p^2.q^3=9.{}^5C_3.p^3.q^2$

q = 9p

We have p + q = 1

p + 9p = 1

10 p = 1

$p=\frac{1}{10}$

40. The real value of 'α' for which $\frac{1-i\sin\alpha}{1+2i\sin\alpha}$ is purely real is

(A) $(n+1)\frac{\pi}{2}$, $n\in\mathbb{N}$

(B) $(2n+1)\frac{\pi}{2}$, $n\in\mathbb{N}$

(C) $n\pi$, $n\in\mathbb{N}$

(D) $(2n-1)\frac{\pi}{2}$, $n\in\mathbb{N}$

Ans. (C)

Sol. $I=\frac{1-i\sin\alpha}{1+2i\sin\alpha}$ to be purely real

Im (z) = 0

1 (2 sin α) – (– sin α) (1) = 0

2 sin α + sin α = 0

3 sin α = 0

Sin α = 0

$\alpha=n\pi;\ n\in\mathbb{N}$

41. The length of a rectangle is five times the breadth. If the minimum perimeter of the rectangle is 180 cm, then

(A) Breadth $\le$ 15 cm

(B) Breadth $\ge$ 15 cm

(C) Length $\le$ 15 cm

(D) Length =15 cm

Ans. (B)

Sol. Given $\ell=5(b)$

Where $\ell=$ length of Rectangle

b = breadth

perimeter (p) = $2(\ell+b)$

= 2(5b + b)

= 2 (6b)

= 12 b

perimeter $\ge$ 180

$12b\ge180$

$b\ge\frac{80}{12}=\frac{30}{2}=15$

$b\ge15$

42. The value of

${}^{49}C_3+{}^{48}C_3+{}^{47}C_3+{}^{46}C_3+{}^{45}C_3+{}^{45}C_4$ is

(A) ${}^{50}C_4$ (B) ${}^{50}C_3$

(C) ${}^{50}C_2$ (D) ${}^{50}C_1$

Ans. (A)

Sol. $= {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{45}C_3 + {}^{45}C_3 + {}^{45}C_4$

$= {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{46}C_4$

$= {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{47}C_4 \quad \left(\because {}^{n}C_r + {}^{n}C_{r-1} = {}^{(n+1)}C_r\right)$

$= {}^{49}C_3 + {}^{48}C_3 + {}^{48}C_4$

$= {}^{49}C_3 + {}^{49}C_4$

$= {}^{50}C_4$

43. Two finite sets have m and n elements respectively. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n respectively

(A) 7,6 (B) 5,1
(C) 6,3 (D) 8,7

Ans. (C)

Sol. $n(A) = m$

$n(B) = n$

$2^{n(A)} = 56 + 2^{n(B)}$

$2^m = 56 + 2^n$

$2^m - 2^n = 56 = 2^6 - 2^3$

$m = 6$ and $n = 3$

44. If $[x]^2-5[x]+6=0$, where [x] denotes the greatest integer function, then

(A) $x \in [3,4]$ (B) $x \in [2,4)$
(C) $x \in [2,3]$ (D) $x \in (2,3]$

Ans. (B)

Sol. $[x]^2 - 5[x] + 6 = 0$

Let $K = [x]$

$K^2 - 5K + 6 = 0$

$(K-2)(K-3) = 0$

$K = 2,3$

$[x] = 2 \Rightarrow x \in [2,3)$

$[x] = 3 \Rightarrow x \in [3,4)$

$\therefore x \in [2,3) U [3,4) = [2,4]$

45. If in two circles, arcs of the same length subtend angles 30° and 78° at the centre, then the ratio of their radii is

(A) $\frac{5}{13}$

(B) $\frac{13}{5}$

(C) $\frac{13}{4}$

(D) $\frac{4}{13}$

Ans. (B)

Sol. $l = r_1\theta_1 = r_2\theta_2$

$\frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{78}{30} = \frac{39}{15} = \frac{13}{5}$

46. If Δ ABC is right angled at C, then the value of tan A + tan B is

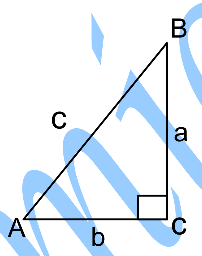
(A) a + b (B) $\frac{a^2}{bc}$

(C) $\frac{c^2}{ab}$ (D) $\frac{b^2}{ac}$

Ans. (C)

Sol. $A + B = 90^0$

By Pyth theorem $a^2 + b^2 = c^2$



$\text{Tan} A = \frac{a}{b}$

$\text{Tan} B = \frac{b}{a}$

$\text{Tan} A + \text{Tan} B = \frac{a}{b} + \frac{b}{a} = \frac{a^2+b^2}{ab} = \frac{c^2}{ab}$

47. In the expansion of $(1 + x)^n$

$\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + ... + n\frac{C_n}{C_{n-1}}$ is equal to

(A) $\frac{n(n+1)}{2}$

(B) $\frac{n}{2}$

(C) $\frac{n+1}{2}$

(D) 3n (n + 1)

Ans. (A)

Sol. $n + 2\left(\frac{n-1}{2}\right) + 3\left(\frac{n-2}{3}\right) + ... + n\left(\frac{1}{n}\right)$

$= n + (n-1) + (n-2) + ... + 1$

$\sum n = \frac{n(n+1)}{2}$

48. If S_n stands for sum to n – terms of a G.P. with 'a' as the first term and 'r' as the common ratio then $S_n : S_{2n}$ is

(A) $r^n + 1$

(B) $\frac{1}{r^n+1}$

(C) $r^n - 1$

(D) $\frac{1}{r^n-1}$

Ans. (A)

Sol. $Sn = \frac{a(1-r^n)}{1-r}; S_{2n} = \frac{a(1-r^{2n})}{1-r}$

$$\frac{S_{zn}}{Sn} == \frac{1-r^{2n}}{1-r^n} = \frac{1-(r^n)^2}{1-r^n} = 1+r^n$$

49. If A.M. and G.M. of roots of a quadratic equation are 5 and 4 respectively, then the quadratic equation is
(A) $x^2 - 10x - 16 = 0$
(B) $x^2 + 10x + 16 = 0$
(C) $x^2 + 10x - 16 = 0$
(D) $x^2 - 10x + 16 = 0$

Ans. (D)

Sol. $\frac{a+b}{2} = 5;\ \sqrt{ab} = 4$

$a + b = 10,\ ab = 16$

$\therefore$ Req. equation is $x^2 - 10x + 16 = 0$

50. The angle between the line $x + y = 3$ and the line joining the points (1, 1) and (–3, 4) is
(A) $\tan^{-1}(7)$
(B) $\tan^{-1}\left(-\frac{1}{7}\right)$
(C) $\tan^{-1}\left(\frac{1}{7}\right)$
(D) $\tan^{-1}\left(\frac{2}{7}\right)$

Ans. (B)

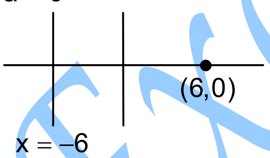
Sol. $m_1 = -1,\quad m_2 = -\frac{3}{4}$

$$\therefore \tan\theta = \frac{-1+\frac{3}{4}}{1+\frac{3}{4}} = \frac{-1}{7} \Rightarrow \theta = \tan^{-1}\left(\frac{-1}{7}\right)$$

51. The equation of parabola whose focus is (6, 0) and directrix is $x = -6$ is
(A) $y^2 = 24x$ (B) $y^2 = -24x$
(C) $x^2 = 24y$ (D) $x^2 = -24y$

Ans. (A)

Sol. a = 6



Eq. (i) $y^2 = 4ax = 4(6)x$

$y^2 = 24x$

52. $\lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}$ is equal to
(A) 2 (B) $\sqrt{2}$
(C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

Ans. (A)

Sol. $\operatorname{lt}_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}$

$$= \operatorname{lt}_{x \to \frac{\pi}{4}} \frac{-\sqrt{2}\sin x}{-\operatorname{cosec}^2 x} = \frac{1}{+\left(\frac{1}{\sqrt{2}}\right)^2} = 2$$

53. The negation of the statement
"For every real number x; $x^2 + 5$ is positive"
(A) For every real number x; $x^2 + 5$ is not positive
(B) For every real number x; $x^2 + 5$ is negative
(C) There exists at least one real number x such that $x^2 + 5$ is not positive
(D) There exists at least one real number x such that $x^2 + 5$ is positive

Ans. (C)

Sol. Conceptual

54. Let a, b, c, d and e be the observations with mean m and standard deviation S. The standard deviation of the observations a + k, b + k, c + k, d + k and e + k is
(A) kS (B) S + k
(C) $\frac{S}{k}$ (D) S

Ans. (D)

Sol. x: a, b, c, d, e

$E[x] = m$

$\sigma_x = S.D(x) = S \Rightarrow \sigma_x{}^2 = S^2$

New data: y: a + K, b + K, c + K, d + K, e + K

$\sigma_y{}^2 = v[y] = v = [x+K] = v[x] = S^2$

$\therefore \sigma_y = S$

55. Let $f : R \to R$ be given by $f(x) = \tan x$. Then $f^{-1}(1)$ is
(A) $\frac{\pi}{4}$
(B) $\left\{n\pi + \frac{\pi}{4} : n \in Z\right\}$
(C) $\frac{\pi}{3}$
(D) $\left\{n\pi + \frac{\pi}{3} : n \in Z\right\}$

Ans. (A)

$f(x) = \tan x, \Rightarrow f^{-1}(x) = \tan^{-1}(x)$

$\Rightarrow f^{-1}(1) = \tan^{-1}(1) = \frac{\pi}{4}$

56. Let $f : R \to R$ be defined by $f(x) = x^2 + 1$. Then the pre images of 17 and –3 respectively are
(A) $\phi, \{4, -4\}$
(B) $\{3, -3\}, \phi$
(C) $\{4, -4\}, \phi$
(D) $\{4, -4\}, \{2, -2\}$

Ans. (C)

Sol. $f(x) = x^2 + 1$

$f^{-1}(x) = \pm\sqrt{x-1}$

$f^{-1}(17) = \pm\sqrt{17-1} = \pm 4$

$f^{-1}(-3) = \pm\sqrt{-3-1} \notin \mathbb{R}$

57. If $A = \begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix}$ and $B = \begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$, then $\frac{dB}{dx}$ is

(A) 3A
(B) –3B
(C) 3B + 1
(D) 1 – 3A

Ans. (A)

Sol. $A = \begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix} = x^2 - 1$

$B = \begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$

$\frac{dB}{dx} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix} + \begin{vmatrix} x & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & x \end{vmatrix} + \begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 0 & 0 & 1 \end{vmatrix}$

$= (x^2 - 1) + (x^2 - 1) + (x^2 - 1)$
$= 3(x^2 - 1)$
$= 3A$

58. Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x & 2x \\ \sin x & x & x \end{vmatrix}$. then $\lim_{x\to 0}\frac{f(x)}{x^2} =$

(A) –1
(B) 0
(C) 3
(D) 2

Ans. (B)

Sol. $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x & 2x \\ \sin x & x & x \end{vmatrix}$

$= \cos x\,[x^2 - 2x^2] - x[2x\sin x - 2x\sin x] + 1\,[2x\sin x - x\sin x]$

$= \cos x(-x^2) - 0 + x\sin x$
$= -x^2\cos x + x\sin x$
$= x\sin x - x^2\cos x$

$\lim_{x\to 0}\frac{f(x)}{x^2} = \lim_{x\to 0}\frac{x\sin x}{x^2} - \frac{x^2\cos x}{x^2}$

$= 1 - 1 = 0$

59. Which one of the following observations is correct for the features of logarithm function to any base b > 1?

(A) The domain of the logarithm functions is R, then set of real numbers
(B) The range of the logarithm function is R^+, then set of all positive real numbers.
(C) The point (1, 0) is always on the graph of the logarithm function
(D) The graph of the logarithm function is decreasing as we move from left to right

Ans. (C)

Sol. $f(x) = \log_b x$ where b > 1

Domain $= (0, \infty) = IR^+$

Range $= (-\infty, \infty) = IR$

$f(x) = \log_b x$ is increasing and point (1, 0) lie on the graph always

60. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3 × 3 matrix A and |A| = 4, then α is equal to

(A) 4
(B) 5
(C) 11
(D) 0

Ans. (C)

Sol. $P = \begin{pmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{pmatrix} = \text{Adj}A$ and |A| = 4

$|\text{Adj}A| = |A|^2$

$1(12 - 12) - \alpha(4 - 6) + 3(4 - 6) = (4)^2$

$-\alpha(-2) + 3(-2) = 16$

$2\alpha - 6 = 16$

$2\alpha = 16 + 6 = 22$

$\alpha = \frac{22}{2} = 11$